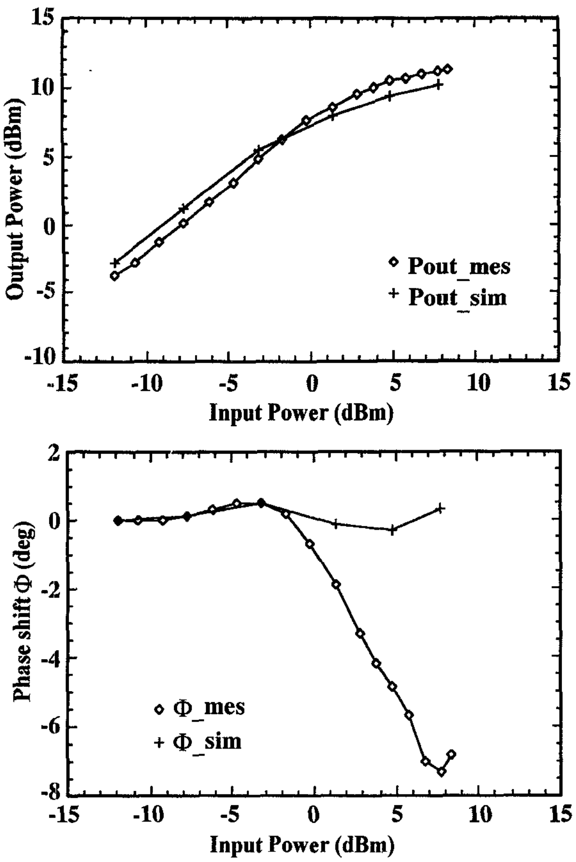

Fig. 6. Measured output power and phase shift versus input power of a low phase distortion MMIC limiter.

gate-source capacitance $Cgs(Vgs, Vds)$ measurements and current-voltage characteristics measured under pulsed conditions.

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Breakdown in the Inhomogeneous Electric Field of a Microwave Transmit-Receive Switch

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***Abstract*—A detailed analytical investigation is made of the threshold for breakdown in microwave transmit-receive switches. The geometry of the keep alive contacts of the switch is modelled as a double cone configuration and the subsequent diffusion equation for the electron density in the presence of strongly inhomogeneous ionization is solved analytically. Predictions for the breakdown power are found to be in agreement with previously presented experimental results.**

I. INTRODUCTION

Breakdown of gases in non-uniform microwave fields has a number of important applications ranging from breakdown in microwave cavity modes [1], [2] to breakdown in the strongly inhomogeneous near fields of antennas [3], [4] and around conducting obstacles in wave guides or cavities [5]–[7]. In the presence of a spatially inhomogeneous microwave field, the concomitant ionization of the gas varies with position and the corresponding diffusion equation determining the break-down properties of the gas becomes complicated. This is particularly so when the field localization region is much smaller than the ionization region, cf. [8]. An interesting and technically important example of this situation occurs in microwave transmit-receive (TR) switches where the strongly localized electric field enhancement created by the sharp conical keep alive contacts significanly lower the power level needed to initiate the breakdown process, cf. [9].

A consistent determination of the breakdown level in the TR-switch is made difficult by two complicating factors: (i) the details of the spatial variation of the electric field enhancement in the vicinity of the contacts are not known and (ii) the geometry of the diffusion process makes the determination of the characteristic diffusion length difficult. In [9], a rough estimate of the average field enhancement and the effective diffusion length were made and the predicted breakdown level could be made to agree with experimental results. In the present work we will give a more detailed and self consistent analysis of the breakdown threshold for TR-switches by analyzing an idealized problem which models the main features of the switch, viz. the spatially varying ionization and the conical diffusion geometry.

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Microwave breakdown is most conveniently analyzed in terms of the equation of continuity for the electron density, n:

$$\frac{\partial n}{\partial t} = (\nu_i - \nu_l)n \quad (1)$$

where ν_i and ν_l denote ionization and loss frequencies respectively. The ionization frequency, ν_i, depends strongly on the strength of the electric field, E. This dependence can be modelled in several convenient ways, e.g. for moderate values of E:

$$\nu_i \cong \nu_o \left(\frac{E}{E_o}\right)^{2\beta} \quad (2)$$

where E_o is a suitable normalization field and β is a parameter of the order of unity. At higher values of E, the ionization frequency saturates and finally even starts to decrease. The loss frequency, ν_l, in the case of the TR-switch, is dominated by diffusional losses which implies that

$$-\nu_i n = D\nabla^2 n \quad (3)$$

where D is the diffusion constant of the electrons, which is only weakly dependent on the electric field and will be approximated as a constant.

As our model problem for the investigation of breakdown in TR-switches we consider a spherical situation where the geometry of the keep alive contacts has been idealized as a double cone configuration. The spatial dependence of the ionization frequency is approximated as

$$\nu_i = \begin{cases} \nu_0 & r < a \\ \nu_0\left(\frac{a}{r}\right)^{2\gamma} & r > a \end{cases} \quad (4)$$

which includes the features of a saturated ionization frequency in a spherical region of radius a close to the vertex of the double cone and a decreasing ionization away from this point. The boundary conditions on the electron density are taken as $n(r, \theta_o, \varphi) = 0 = n(r, \pi - \theta_o, \varphi)$, and $n(r, \theta, \varphi) \to 0$ as $r \to \infty$ where (r, θ, φ) are spherical coordinates and $2\theta_o$ is the cone angle.

Assuming solutions which are independent of φ, the breakdown threshold for cw operation is obtained from the lowest order eigenvalue of the equation

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial n}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\cdot\left(\sin\theta\frac{\partial n}{\partial\theta}\right) + \frac{\nu_i(r)}{D}n = 0 \quad (5)$$

with boundary conditions as given above. Equation (5) can be separated by writing $n(r, \theta) = R(\rho)\phi(\theta)$, where $\rho = r/a$. The corresponding equations are

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\phi}{d\theta}\right) + k\phi = 0 \quad (6a)$$

$$\frac{1}{\rho^2}\frac{d}{d\rho}\left(\rho^2\frac{dR}{d\rho}\right) - k\frac{R}{\rho^2} + \frac{\nu_i a^2}{D}R = 0 \quad (6b)$$

where k is the constant of separation and $\nu_i a^2/D$ plays the role of the eigenvalue, which is to be determined.

A. Angular Dependence

Equation (6a) for $\phi(\theta)$ has solutions in terms of Legendre functions, $Q(\cos\theta)$. Using the substitution; $\cos\theta = z, \phi(\theta) = Q(\cos\theta) = Q(z), k = \nu(\nu+1)$, and (7a) can be written

$$(1 - z^2)\frac{d^2Q}{dz^2} - 2z\frac{d\theta}{dz} + \nu(\nu+1)Q = 0. \quad (7)$$

The condition that $Q > 0$ for $|z| < z_o = \cos\theta_o$ and $Q(z_o) = 0$ determines ν and consequently k. Although an explicit solution of this problem is not possible, the solutions corresponding to three particular cases give a good overview of the properties of the general solution:

(i) When $\nu \to 0\,(\theta_o \to 0)$ we obtain $k = \nu(\nu+1) \simeq 1/\ln(1/\theta_o)$ and $\phi(\theta) \cong 1 - \ln(\sin\theta)/\ln(\sin\theta_o)$. (ii) When $\nu = 1\,(\cos\theta_o \simeq 0.83, \theta_o \simeq 35°)$ we obtain $k = 2$ and $\phi(\theta) = Q_l(\cos\theta)$ where $Q_l(x)$ is the Legendre function of the second kind with index 1. (iii) When $\nu = 2\,(\cos\theta_o \simeq 0.58, \theta_o \simeq 55°)$ we obtain $k = 6$ and $\phi(\theta) = -P_2(\cos\theta) = \frac{1}{2}(1 - 3\cos^2\theta)$ where $P_2(x)$ is the Legendre function of the first kind with index 2.

B. Radial Dependence

Equation (6b) for the radial dependence corresponds to Bessel equations for $r < a$ as well as for $r > a$. The solutions compatible with n being finite as $\rho \to 0$ and vanishing as $\rho \to \infty$ are

$$R(\rho) \propto \begin{cases} \frac{1}{\sqrt{\rho}}J_\mu(\eta\rho) & \rho < 1 \\ \frac{1}{\sqrt{\rho}}J_\delta\left(\frac{\eta}{|\epsilon|}\rho^{-\epsilon}\right) & \rho > 1 \end{cases} \quad (8)$$

where $\eta^2 = \nu_o a^2/D, \mu = (k + 1/4)^{1/2} = \nu + 1/2, \delta = \mu/|\epsilon|$, and $\epsilon = \gamma - 1$.

The breakdown threshold, i.e. the parameter η is determined as the smallest value which satisfies the requirement that $R(\rho)$ and $dR/d\rho$ are continuous at $\rho = 1$. This yields the following transcendental equation for η:

$$\frac{J'_\mu(\eta)}{J_\mu(\eta)} = -\text{sign}\,\epsilon\frac{J'_\delta\left(\frac{\eta}{|\epsilon|}\right)}{J_\delta\left(\frac{\eta}{|\epsilon|}\right)} \quad (9)$$

where prime denotes differentiation with respect to the argument. Again, a general solution of (9) is not possible but good insight into the behavior of the solution can be obtained from the particular cases:

(i) For $\gamma < 1$, which implies a very slow drop of ionization frequency with distance, we have $\epsilon < 0$ and (9) implies $\eta = 0$, i.e. breakdown occurs for arbitrarily small values of ν_o. (ii) For $\gamma \to 1^+$, we obtain $\epsilon \to 0^+$, and $\eta/|\epsilon| \to j'_{\mu/|\epsilon|,1} \simeq \mu/|\epsilon|$, where $j'_{\mu/|\epsilon|,1}$ denotes the first zero of $J'_{\mu/|\epsilon|}(x)$. This yields $\eta \to \mu = \nu + 1/2$. Corresponding to the three different cone angles, θ_o, investigated under (a) we obtain

$$\eta = \begin{cases} \frac{1}{2} & \theta_o \to 0° \\ \frac{3}{2} & \theta_o \cong 35° \\ \frac{5}{2} & \theta_o \cong 55°. \end{cases} \quad (10)$$

(iii) For $\gamma = 2$ we obtain $\epsilon = 1$ and (9) reduces to $J_\mu(\eta) = 0$. In particular

$$\eta \cong \begin{cases} 1.2 & \theta_o \to 0 \\ 2.5 & \theta_o \cong 35° \\ 4.0 & \theta_o \cong 55°. \end{cases} \quad (11)$$

(iv) For $\gamma \to \infty$, which implies a very rapid drop of ionization with radius, we have $\epsilon \to \infty$ and (9) is reduced to

$$\eta\frac{J'_\mu(\eta)}{J_\mu(\eta)} = -\mu \quad (12)$$

with the solutions

$$\eta \cong \begin{cases} \pi/2 & \theta_o = 0 \\ \pi & \theta_o \cong 35° \\ 4.5 & \theta_o \cong 55°. \end{cases} \quad (13)$$

The general solution of the transcendental equation, (9), which determines the breakdown threshold, cannot be given explicitly. However the approximate solutions give a clear picture of the dependence of η on the parameter θ_o, which characterizes the cone angle and γ, which determines the fall off of the ionization with radius.

It is seen that η is not a sensitive function of these parameters. The geometry of the keep-alive contacts implies that θ_o tends to be rather small, typically $\theta_o \cong 20°$. The proper choice of γ can be estimated as follows: The electric field in the presence of a conical disturbance can be found as, [10]:

$$E \simeq \frac{E_o}{k_o r \sin\theta \ln(\cot(\theta_o/2))} \tag{14}$$

where k_o is the vacuum wave-number and E_o is the amplitude of the incident plane wave. As an average value of E we will use $E \simeq E_o/(k_o r)$. Furthermore, assuming $\nu_i \sim E^2$, cf. (2) and [9], we obtain $\nu_i = \alpha E^2 \equiv \nu_o a^2/r^2$, which implies $\nu_o a^2 = \alpha E_o^2/k_o^2$ and $\gamma = 1^+$.

The breakdown condition for $\theta_o = 20°$ and $\gamma = 1^+$ is then $\eta \approx 1$, i.e.

$$\frac{\nu_o a^2}{D} \simeq 1 \tag{15}$$

In fact, for an electric field given by (14), the ionization frequency depends on the angle θ as well as on the radius, r. For the case $\nu_i \sim E^2$, the ionization frequency becomes

$$\nu_i = \alpha E^2 = \frac{\alpha E_o^2}{k_o^2 \ln^2\left(\cot\frac{\theta_o}{2}\right)} \frac{1}{r^2 \sin^2\theta} \equiv \nu_o \frac{a^2}{r^2 \sin^2\theta} \tag{16}$$

The corresponding diffusion equation can be solved exactly to yield the eigenvalue

$$\eta = \frac{\pi}{2\left|\ln\tan\frac{\theta_o}{2}\right|} \cong 1 \tag{17}$$

in accordance with our previous estimate, (15). Thus, using (15) the following breakdown condition is obtained:

$$E_o \cong \left(\frac{k_o^2 D}{\alpha}\right)^{1/2} \tag{18}$$

which does not depend on the characteristic radius a.

In the experiments presented in [9], $k_o \simeq 2\,\text{cm}^{-1}, \alpha \simeq 2 \cdot 10^2\,(\text{V/cm})^{-2}\,\text{s}^{-1}$ and D can be estimated as $D \simeq 10^5\,\text{cm}^2/\text{s}$, which implies $E_o \simeq 45$ V/cm. This corresponds to a power level of 2W at breakdown in good agreement with the observed level (1W). Furthermore, the characteristic distance, a, at which the ionization rate (and the field) saturates should be of the order of the distance between the truncated keep alive contacts (2a) in the TR-switch, i.e. $a \simeq 0.1$ mm. The maximum electric field enhancement is then obtained from (14) as $E/E_o \simeq 30$, again in good agreement with previous estimates, [9], [11].

The present analysis has considered in detail the problem of determining the threshold for breakdown in the strongly inhomogeneous electric field characteristic of microwave TR switches. The diffusion equation for the electron density has been solved exactly in the geometry of a double cone, which closely models the configuration of the breakdown region around the keep alive contacts in a TR switch. The analysis provides a significant step towards a self consistent determination of the breakdown threshold in microwave TR-switches. The predicted power levels for breakdown are shown to be in good agreement with previously published experimental results.

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Impedance, Attenuation and Power-Handling Characteristics of Double L-Septa Waveguides

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***Abstract*—Rectangular waveguides with two L-shaped septa attached to the broad walls in antipodal configuration have been shown theoretically to be an improved variant of the Double T-Septa Guide having larger cut off wavelength and broader bandwidth of the dominant TE mode [9]. Numerical data on the attenuation, impedance and power handling capability of this new type of broadband guides are now presented here as additional design information.**

I. Introduction

Recently we have proposed rectangular waveguides with shaped septa as new broadband transmission lines. The proposals were based on the results obtained from rigorous theoretical analysis of certain structures conceived intuitively. The first type of such guides has one or two T-shaped septa instead of the conventional ridges (Fig. 1(a)) to provide the necessary capacitive loading for increasing the modal separation [1]–[6]. Enhancement of bandwidth by dielectric loading of the septa gap was also determined theoretically [7], [8]. In the second type of proposed guides, the two septa are L-shaped and are attached to the broad walls in antipodal configuration (Fig. 1(b)). It has been designated the Double L-Septa Guide (DLSG). The results

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